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Finite particle limit in the Ensemble Kalman Sampler

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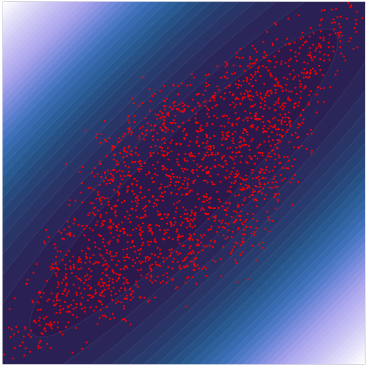

Figure: 100 particles using $\Delta t = 10^{-3}$ following **overdamped Langevin** in an anisotropic potential.

Sampling a **highly anisotropic** Boltzmann distribution $x \mapsto \mathrm{e}^{-V(x)}$

Overdamped Langevin dynamics:

$$\mathrm{d}X_t = -\nabla V(X_t)\,\mathrm{d}t + \sqrt{2}\,\mathrm{d}W_t$$

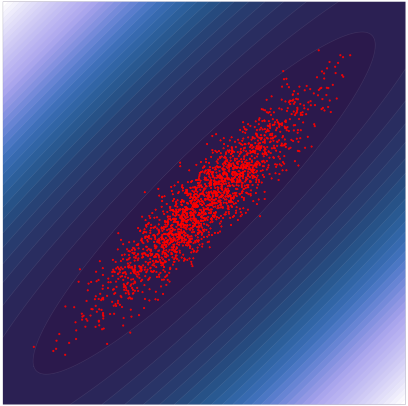

Figure: 100 particles using $\Delta t = 10^{-3}$ following **preconditioned Langevin** in an anisotropic potential.

Sampling a **highly anisotropic** Boltzmann distribution $x \mapsto \mathrm{e}^{-V(x)}$

Overdamped Langevin dynamics:

$$\mathrm{d}X_t = -\nabla V(X_t)\,\mathrm{d}t + \sqrt{2}\,\mathrm{d}W_t$$

Preconditioned Langevin dynamics:

$$\mathrm{d}X_t = -\mathcal{C}_V \nabla V(X_t)\,\mathrm{d}t + \sqrt{2\mathcal{C}_V}\,\mathrm{d}W_t$$

where $\mathcal{C}_V$ is the covariance matrix of the target distribution.

Ensemble Kalman Sampler (EKS)

$$\mathrm{d}X_t^n = -\mathcal{C}(\rho_{X_t})\nabla V(X_t^n)\,\mathrm{d}t + \frac{d+1}{N}(X_t^n - \mathcal{M}_{X_t}) + \sqrt{2\mathcal{C}(\rho_{X_t})}\,\mathrm{d}W_t^n$$

with $\mathcal{C}(\rho_{X_t})$ the empirical covariance of the finite particle ensemble
and correction term for **finite number** of particles

- While particles are **interacting** through $\mathcal{C}(\rho_{X_t})$, the invariant measure is as if they were independent $\mathrm{e}^{-V(x)} \otimes \cdots \otimes \mathrm{e}^{-V(x)}$
- **Affine invariant** method, always well-conditioned
- **Gradient-free** approximation can be used
- **Square root approximation** of covariance for high-dimension if needed

Finite particle and mean-field limit

$$\mathrm{d}X_t^n = -\mathcal{C}(\rho_{X_t})\nabla V(X_t^n)\,\mathrm{d}t + \frac{d+1}{N}(X_t^n - \mathcal{M}_{X_t}) + \sqrt{2\mathcal{C}(\rho_{X_t})}\,\mathrm{d}W_t^n$$

with $\mathcal{C}(\rho_{X_t})$ the empirical covariance of the finite particle ensemble and correction term for **finite number** of particles

Contributions

- Convergence to the mean-field limit $N \to \infty$ (propagation of chaos).
- For finite N, we aim to show uniform in time convergence as well i.e.,

$$\mathbb{E}\left[\sum_{n=1}^{N} |X_t^n - Y_t^n|^2\right] \leq C\mathrm{e}^{-(2+\varepsilon(N))t} \qquad \text{uniformly in time}$$

where Y_t at equilibrium, with a synchronous coupling.